4 MIXED REVISION
CHAPTERS 10 • 11 • 12

Multiple choice

1 If z is a complex number, the solutions to $z^2 - 2z + 10 = 0$ are:

A $z=-1\pm 3i$ B $z=1\pm 3i$ C $z=-1\pm i\sqrt{3}$ D $z=1\pm i\sqrt{3}$ E $z=-1\pm\sqrt{11}$

2 What is the vector for the translation 2 down and 5 to the left?

A $\begin{bmatrix}-2 & 0\\ 0 & 5\end{bmatrix}$ B $\begin{bmatrix}2 & -5\end{bmatrix}$ C $\begin{bmatrix}-5\\ -2\end{bmatrix}$ D $\begin{bmatrix}-5 & 0\\ 0 & -2\end{bmatrix}$ E $\begin{bmatrix}-2\\ -5\end{bmatrix}$

3 The amplitude and period of the graph of $y = 3\sin[2(x - \pi)]$ are respectively

A $3, 2$ B $-3, \frac{1}{2}$ C $3, \frac{\pi}{2}$ D $3, \pi$ E $3, 2\pi$

4 The value of $\frac{1+\sqrt{3}-i\sqrt{3}}{1+i}$ in $a + bi$ form is:

A $-\frac{1}{2}-i\frac{\left(2\sqrt{3}-1\right)}{2}$ B $\frac{1}{2}-i\frac{\left(2\sqrt{3}-1\right)}{2}$ C $\frac{1}{2}-i\frac{\left(2\sqrt{3}+1\right)}{2}$

D $\frac{1}{2}+i\frac{\left(2\sqrt{3}-1\right)}{2}$ E $-\frac{1}{2}-i\frac{\left(2\sqrt{3}+1\right)}{2}$

5 What is the matrix for a reflection in the line $y = 3x$?

A $\begin{bmatrix}3 & 1\\ 1 & -3\end{bmatrix}$ B $\begin{bmatrix}1 & 3\\ 3 & -1\end{bmatrix}$ C $\begin{bmatrix}-8 & 10\\ 10 & 8\end{bmatrix}$ D $\begin{bmatrix}-\frac{4}{5} & \frac{3}{5}\\ \frac{3}{5} & \frac{4}{5}\end{bmatrix}$ E $\begin{bmatrix}\frac{3}{5} & \frac{4}{5}\\ \frac{4}{5} & -\frac{3}{5}\end{bmatrix}$

6 $y = 2\sec(3x)$ can be expressed as

A $y = 2\cos(3x)$ B $y=\frac{2}{\cos(3x)}$ C $y=2\cos\left(\frac{1}{3}x\right)$

D $y=\frac{1}{2\cos(3x)}$ E $y=\frac{1}{2}\cos\left(\frac{1}{3}x\right)$

7 An equation with roots $\frac{-1\pm i\sqrt{3}}{2}$ is:

A $z^2 + z + 1 = 0$ B $z^2 - z + 1 = 0$ C $z^2 - z - 1 = 0$

D $z^2 + z - 1 = 0$ E none of the above

8 What is the area of the image of the rhombus $A(-1, 3)$ $B(1, 7)$ $C(3, 3)$ $D(1, -1)$ under the transformation $(x, y) \to (3x + y)(5y - 4x)$?

A 16 B 112 C 224 D 304 E 608

9 The solution(s) of the equation $\tan(2x)=-\frac{\sqrt{3}}{3}$ for $-\frac{\pi}{2}\leq x\leq\frac{\pi}{2}$ is/are

A $-\frac{\pi}{6}, \frac{5\pi}{6}$ B $-\frac{\pi}{3}, \frac{4\pi}{3}$ C $-\frac{\pi}{6}, \frac{5\pi}{6}$ D $\frac{5\pi}{12}$ E $-\frac{\pi}{12}, \frac{5\pi}{12}$

9780170250276

Short answer

1 Factorise $x^2 - 4x + 5$ over the complex plane.

2 What is the image of $A(-2, 1)$ $B(2, 4)$ $C(7, 2)$ $D(4, -3)$ under a dilation with factors of 3 in the x-direction and 2 in the y-direction?

3 Express $3\cos(\theta) + 2\sin(\theta)$ in the form of $r\cos(\theta - \alpha)$.

4 If $z = 1 + i$ and $w = -2 + 5i$, find in the form $x + yi$:

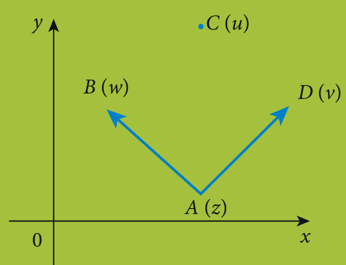
a $z\overline{w}$ b $|z + w|$

5 Find the image of $A(-1, -2)$ $B(2, 3)$ $C(7, 4)$ under a rotation of $-\frac{\pi}{6}$.

6 Sketch the graph of $y = 2\tan\left(\frac{x}{2}\right)$ for $0 < x < 5\pi$.

Application

1 In the diagram, the vertices of a square $ABCD$ are represented by the complex numbers z, w, u and v.



a Find the vector $\overrightarrow{AD}$ in terms of z and v. Find the vector $\overrightarrow{AB}$ in terms of z and w.

b Hence, or otherwise, find the complex number u that represents C in terms of z, w and v.

2 Consider the equation $z^2 + uz + (2 - i) = 0$. If i is a root of the equation, find the complex number u.

3 What is the single transformation that is the result of a reflection in the line through the origin with inclination 50° followed by a rotation of 20°?

4 Use matrices or other methods to prove that dilation is associative under composition. That is, for any dilations **S**, **R** and **T**, $\mathbf{S} \circ (\mathbf{R} \circ \mathbf{T}) = (\mathbf{S} \circ \mathbf{R}) \circ \mathbf{T}$.

5 a Express $\sin(x) + 5\cos(x)$ in the form $r\sin(\theta + \alpha)$.

b Find the general solution for the equation $\sin(x) + 5\cos(x) = -\frac{1}{2}$.

c Use a graph to find the solution(s), in degrees correct to 2 decimal places, of $\sin(x) + 5\cos(x) = -\frac{1}{2}$ for $0° \leq x \leq 360°$.

6 The displacement of a particle is given by $x = 2\sin(3t) - \cos(3t)$, where x is measured in metres and time, t, in seconds.

a Sketch the graph of the displacement to illustrate that the particle follows Simple Harmonic Motion (SHM).

b Find the amplitude and period of the motion.

c Find the greatest distance, correct to 3 decimal places, that the particle moves from its origin.

9780170250276